On the Concept of Brightness in Music

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ABSTRACT

In Music, the concept of Brightness has progressively become a topic of heated debate. According to several authors, when a significant number of Modes is involved in a comparison, it is not possible to exactly establish a brightness hierarchy devoid of inconsistencies and ambiguities. A method to solve the problem is proposed in this paper. Firstly, in order to avoid counter-intuitive solutions, the Brightness of all the Seventh Chords is determined; subsequently, by resorting to a Reference Scale arising from the Overtone Series, all the inconsistencies (e. g. occurrence of Modes yielding the same Absolute Brightness) are removed.

1 Introduction to the paper

In (Modal) Music, the concept of Brightness has progressively become a topic of heated debate. The Brightness of a Mode/Scale is undoubtedly related to the individual perception. [1] The Intervallic Content of a Scale surely influences the perception of Brightness/Darkness: suffice it to think that, at least as far as the Ordinary (Heptatonic) Scales are concerned, the quality of the Mediant (the 3rd Degree) can in itself convey a Bright or Dark Mood, depending on its quality (Major or Minor). Major or Minor (and, in general, Sharpened or Flattened) Intervals suggest, respectively, Brighter or Darker Moods. [2] Therefore, the Brightness of a Mode may be evaluated, to a certain extent, by examining the Intervals (measured with respect to the Tonic) which characterize the Scale. For this purpose, we can take into consideration the so-called Pitch Class Set (which is herein regarded as a Vector, denoted by s) [3,4]. The Pitch Class Set describes all the Real Distances (all the Specific Intervals), in semitones, between any Degree of the Scale and the Tonic. The value of the first Specific Interval of any Set (the value of the first Component of the Vector) is always 0, since the Distance between the Tonic and itself is obviously null. For the *Ionian* Scale, e.g., we have

$$s_{Ion.} = (0,2,4,5,7,9,11)$$

In the light of what previously underlined, an estimation of the Brightness, herein denoted by B, may be carried out by adding up all the Components of the above-mentioned Vector (from now onwards, this value of Brightness will be considered as being Absolute).

$$B_{Ion.} = \sum_{n=2}^{7} s_n = 2+4+5+7+9+11 = 38$$

The subscript n runs from 2 to 7 since the first Component of s is always equal to 0.

This method, however, entails several inconsistencies. E.g., the *IpoIonian* Scale (a Minor Scale) turns out to be paradoxically brighter than the *Mixolydian b6* (a Major Scale):

$$s_{Ipo\ Ion.} = (0,2,3,5,7,9,11)$$

$$B_{Ipo\ Ion.} = \sum_{n=2}^{7} s_n = 2 + 3 + 5 + 7 + 9 + 11 = 37$$

$$s_{Mixo.b6} = (0,2,4,5,6,8,10)$$

$$B_{Mixo.b6} = \sum_{n=2}^{7} s_n = 2 + 4 + 5 + 7 + 8 + 10 = 36$$

$$B_{Ipo\ Ion.} > B_{Mixo.\ b6}$$

Moreover, it is de facto impossible to manage some ambiguities by resorting to the above-mentioned method. *Aeolian* and *Dorian b2*, e.g., are characterized by the same Absolute Brightness:

$$s_{Aeo.} = (0,2,3,5,7,8,10)$$

$$B_{Aeo.} = \sum_{n=2}^{7} s_n = 2 + 3 + 5 + 7 + 8 + 10 = 35$$

$$s_{Dor.b2} = (0,1,3,5,7,9,10)$$

$$B_{Dor.b2} = \sum_{n=2}^{7} s_n = 1 + 3 + 5 + 7 + 9 + 10 = 35$$

$$B_{Aeo.} = B_{Dor.b2}$$

In order to remove inconsistencies and ambiguities, a Reference Scale/Mode is introduced

2 The Reference Scale: Definition and Characterization

Our Reference Scale is the *Lydian Dominant b6* (nothing but a *Lydian Dominant*, i.e. a so-called *Acoustic* Scale, with a flattened 6th). The reason for the choice is quite banal: the above-mentioned Scale (which can be regarded as the 4th Mode of the *Neapolitan Major*, actually a Minor Scale) can be directly obtained from the Overtone Series, by taking all the Harmonics from 8 to 14 (by selecting the first 7 different Harmonics).

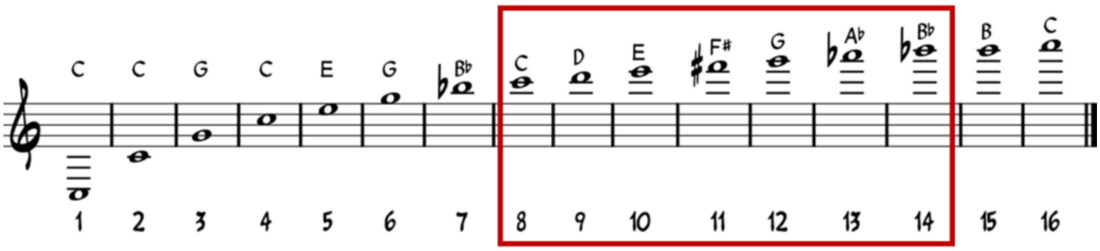

Figure 1. Harmonic Series and Reference Scale.

2.1 Representation by means of the Unit Circle

The 12 Pitches of an Equal Temperament Tuning System con be depicted by resorting to a Unit Circle (the Radius, usually denoted by r, is equal to 1). Any Pitch becomes a Point of the above-mentioned Circle (there are 12 Points altogether); any pair of adjacent Points/Pitches delimits an arc subtending a 30° angle (at the Center).

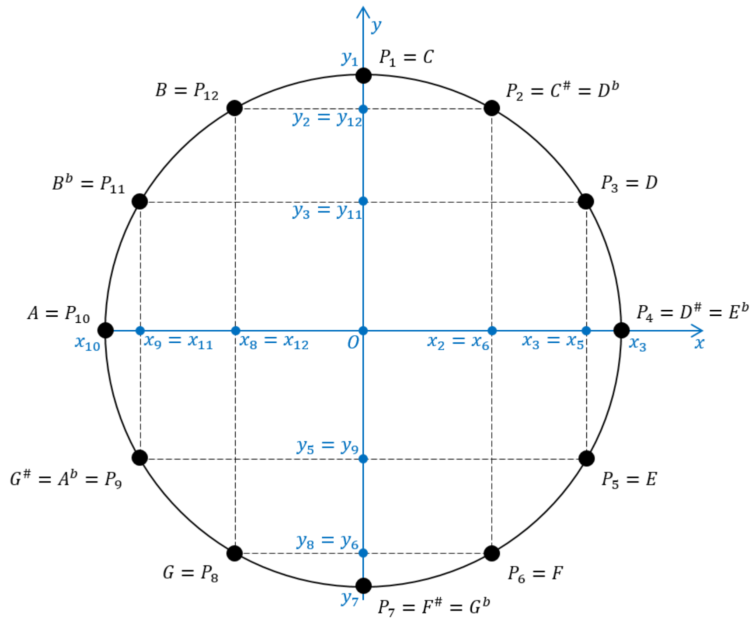

Figure 2. Unit Circle and Tones.

The coordinates of the Points/Pitches can be easily determined by resorting to the fundamental trigonometric functions, e.g.:

$$P_2 \equiv (\cos 60°, \sin 60°)$$

$$P_3 \equiv (\cos 30°, \sin 30°)$$

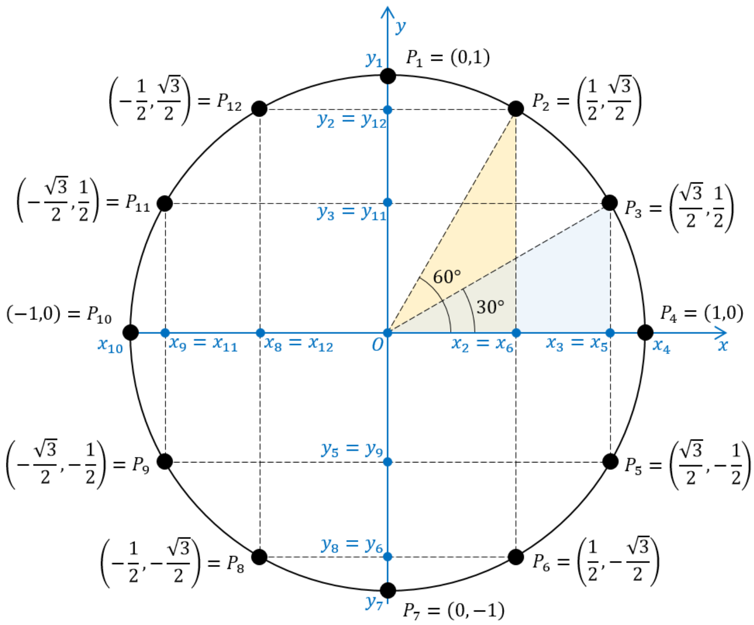

Figure 3. Coordinates.

Any Heptatonic Scale (any Mode characterized by a Cardinality equal to 7) can obviously be described by placing 7 Points/Pitches on the Unit Circle. Our Reference Mode, the *Lydian Dominant b6* Scale, is shown in **Figure 4**.

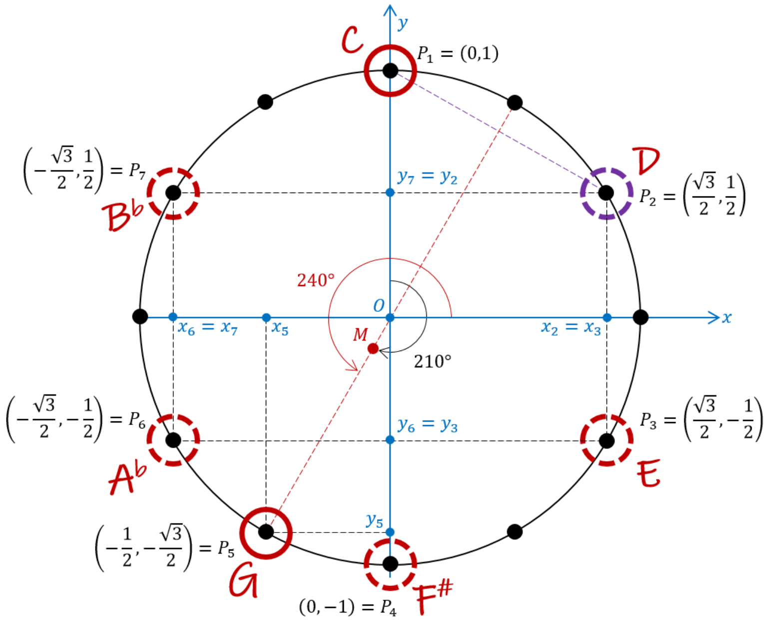

Figure 4. Reference Mode

The *Lydian Dominant b6* Scale is “Dihemitonic”, since there are two pairs of consecutive Degrees separated by a single semitone (4th → 5th and 5th → 6th), and “Cohemitonic”, since the above-mentioned couples are clearly adjacent. According to Forte and Rahan [5,6], the Reference Scale cannot be regarded as a Prime Form (in our case, the Prime Form of the Modal Family is the 7th Mode of the *Neapolitan Major*). The Reference Scale is clearly characterized by 5 Imperfections (a Pitch/Tone is considered as being Imperfect, with respect to a particular Mode, if the corresponding Perfect 5th is not included in the Scale), as highlighted by resorting to the dashed circles.
The Scale is obviously unbalanced. The coordinates (x_M andy_M) of the Center of Gravity can be deduced as follows:

$$x_M = \frac{1}{7}\sum_{i=1}^{7} x_i = \frac{1}{7}\left(0 + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + 0 - \frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) = -\frac{1}{7}\left(\frac{1}{2}\right) = -\frac{1}{14} \cong -0{,}07$$

$$y_M = \frac{1}{7}\sum_{i=1}^{7} y_i = \frac{1}{7}\left(1 + \frac{1}{2} - \frac{1}{2} - 1 - \frac{\sqrt{3}}{2} - \frac{1}{2} + \frac{1}{2}\right) = -\frac{1}{7}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{14} \cong -0{,}12$$

As for the Distance between the Center of Gravity and the Center of the Unit Circle, we can write:

$$\overline{OM} = \sqrt{(x_M - x_O)^2 + (y_M - y_O)^2} = \sqrt{(x_M)^2 + (y_M)^2} \cong 0{,}14$$

Since r = 1, the value just deduced equates the Eccentricity:

$$e = \frac{\overline{OM}}{r} = \frac{0{,}14}{1} = 0{,}14$$

The Center of Gravity is placed along the Symmetry Axes (the Reference Scale is clearly symmetric):

$$\tan^{-1}\left(\frac{y_M}{x_M}\right) = \tan^{-1}\left(\frac{-\frac{\sqrt{3}}{14}}{-\frac{1}{14}}\right) = \tan^{-1}(\sqrt{3}) = 240°$$

The Angle between the x Axes and the Symmetry Axes (measured counter-clockwise) is equal to 240°. Consequently, the Angle between the y Axes and the Symmetry Axes (measured clockwise, therefore starting from the Tonic) is equal to 210°.

2.2 Absolute Brightness

For the Reference Scale the Pitch Class Set acquires the following form:

$$s_{Lyd.\ Dom.b6} = (0,2,4,6,7,8,10)$$

Consequently, for the Absolute Brightness we can write:

$$B_{Lyd.\ Dom.b6} = \sum_{n=2}^{7} s_n = 2+4+6+7+8+10 = 37$$

2.3 Intervallic Structure

The Intervallic Structure of a Scale/Mode [7] can be described by resorting to a Vector, herein denoted by i_s . The number of Components of this Vector is equal to the Cardinality (the number of Pitches/Tones) of the Scale. The Components of i_s represent, progressively, the Specific Intervals (in semitones) between any Pitch/Tone and the subsequent one, starting from the Tonic. In our case, we have:

$$i_s = (2,2,2,1,1,2,2)$$

2.4 Interval Vector, Spectrum, Spectral Distribution, Evenness, Spectral Variation

The Interval Vector, which describes the Intervallic Content of a Scale, can actually be regarded as a Spectrum. The Components of the above-mentioned Vector, in fact, convey the number of occurrences of each Specific Interval, starting from the shortest (*Minor 2nd*, *Major 2nd*, *Minor 3rd*, *Major 3rd*, *Tritone*, *Perfect 5th*). Any Specific Interval is obviously considered as being equivalent to its inverse (as a consequence, any Scale includes, e.g., an equal number of *Major 2nd* and *Minor 7th* Intervals). Denoting with S the Interval Vector, in our case, we have:

$$S = (2,6,2,6,2,3)$$

In other terms, there are two *Minor 2nd* Intervals, six *Major 2nd* Intervals, two *Minor 3rd* Intervals, six *Major 3rd* Intervals, two *Tritones* and three *Perfect 5th* Intervals. According to Hanson [8], the Interval Vector of the Reference Scale may take the form $p^2m^6n^2s^6d^2t^3$, where p denotes a *Perfect 5th* (or, equivalently, a *Perfect 4th*), m a *Major 3rd* (or, equivalently, a *Minor 6th*), n a *Minor 3rd* (or, equivalently, a *Major 6th*), s a *Major 2nd* (or, equivalently, a *Minor 7th*), d a *Minor 2nd* (or, equivalently, a *Major 7th*), t a *Tritone* (the Specific Intervals are ordered from the most consonant to the most dissonant). As a consequence, Hanson's Spectrum [8], denoted by S_H, acquires the following form:

$$S_H = p^2m^6n^2s^6d^2t^3$$

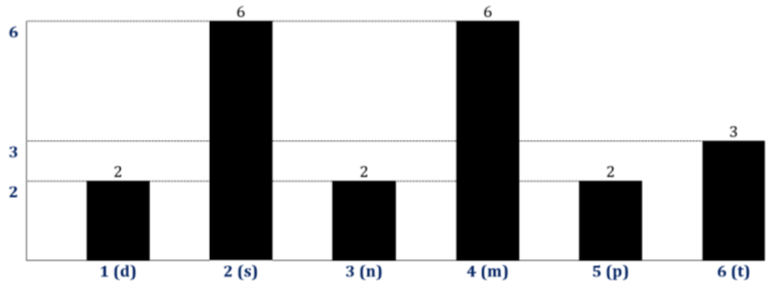

Figure 5. Reference Mode: Spectrum.

Any Diatonic/Generic Interval is characterized by its own Spectrum, herein denoted by $\langle i \rangle$, where i represents the Number of Steps, from the lowest Pitch/Tone, in terms of Scale Degrees (e.g., $\langle 1 \rangle$ denotes a Diatonic/Generic *2nd* Interval, $\langle 2 \rangle$ a Diatonic/Generic *3rd* Interval). A Specific Interval clearly represents the width (in semitones) of a Diatonic/Generic Interval: for the Ionian Mode, e.g., $\langle i \rangle = \{1,2\}$, since the Scale admits Diatonic/Generic *2nd* Intervals characterized by a width equal to 1 (between Mediant and Subdominant, and between Leading Tone and Tonic) and Diatonic/Generic *2nd* Intervals characterized by a width equal to 2 (between the remaining pairs of adjacent Tones).

In **Figure 6** a representation of the Spectral Distribution for the Reference Scale is provided.

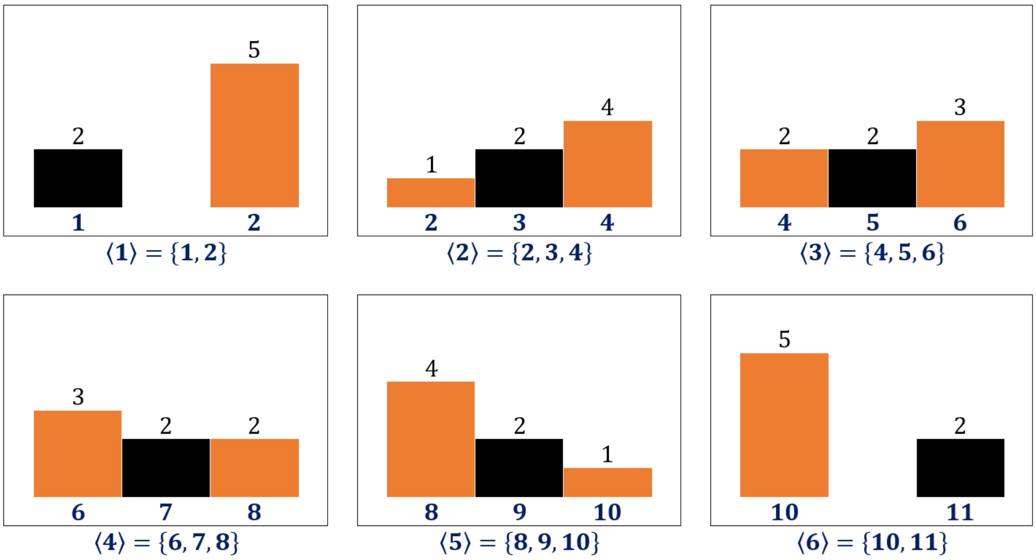

Figure 6. Reference Mode: Spectral Distribution

In general, if all the (Diatonic) Spectra consist of a single Specific Interval (e.g. see the *Whole-Tone* Scale), the Scale is considered as being Equally Distributed. Obviously, there are no Heptatonic Scales Equally Distributed (the Cardinality, 7 in this specific case, is an odd number). In a Maximally Even Scale [9] all the Spectra are characterized by 2 components, and the difference between the values of the Specific Intervals is no greater than 1 (e.g. the Ionian Scale is Maximally Even).
As a consequence, the Reference Scale in not Maximally Even.

The Spectrum Width, herein denoted by $w\langle i \rangle$, is equal to the difference between the maximum and minimum value of the Spectral Components (the difference between the highest and lowest Specific Interval). The Spectrum Variation, herein denoted by $\bar{w}$, is nothing but the sum of all the Spectrum Widths, divided by the number of Pitches/Tones (the Cardinality of the Scale). High value of Spectrum Variation entails low Evenness (obviously, the Spectrum Variation of any Equally Distributed Scale is equal to 0). For the Reference Scale we can write:

$$w\langle i \rangle = max\langle i \rangle - min\langle i \rangle$$

$$\bar{w} = \frac{1}{7}\sum_{i=1}^{6} w\langle i \rangle = \frac{(2-1)+(4-2)+(6-4)+(8-6)+(10-8)+(11-10)}{7} = \frac{1+2+2+2+2+1}{7} = \frac{10}{7} \cong 1{,}43$$

2.5 Propriety

The Propriety of a Scale is related to the concept of Ambiguity between Generic and Specific Intervals.

According to Rothenberg [10], a Scale/Mode can be considered as being Proper if there are no overlaps in its Spectral Distribution (although a Specific Interval can describe two different Diatonic/Generic Intervals).

The Reference Scale is evidently even.

The *Neapolitan Minor* Scale, e.g., is not Proper: in this Mode, in fact, a Diatonic 3rd can be smaller (in terms of semitones) than a Diatonic 2nd, and a Diatonic 7th can be smaller than a Diatonic 6th. This condition can be described as follows:

$$\exists i \in N, 1 < i < 6 \;\; s.t. \;\; max\langle i \rangle > min\langle i+1 \rangle$$

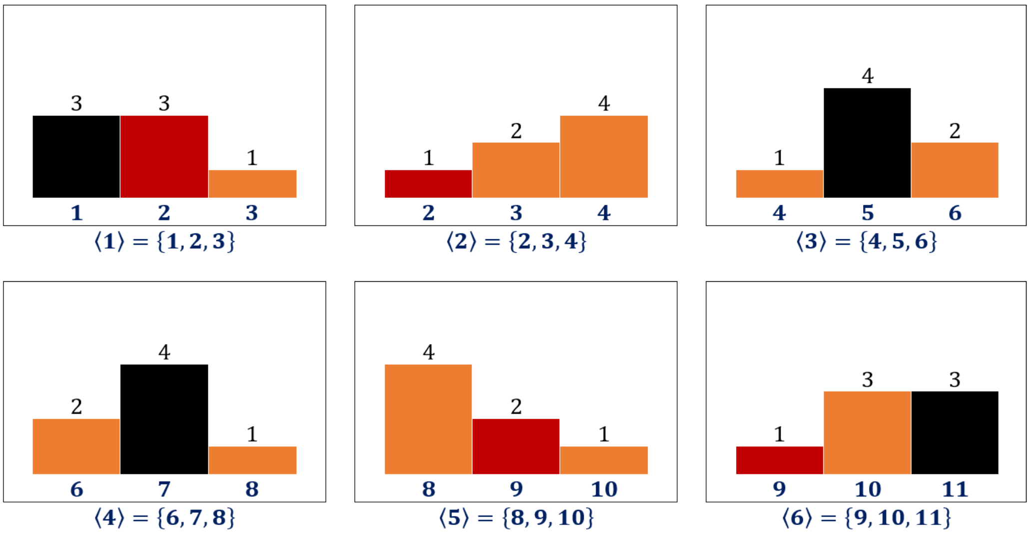

Figure 7. Neapolitan Minor: Spectral Distribution.

3 The Method

Any Mode, obviously, entails a Seventh Chord built on the Tonic of the Scale (the Root of the Chord). The first step consists in establishing a Brightness hierarchy between Modes producing identical Seventh Chords: in order to accomplish the task, we carry out a direct comparison between the above-mentioned Modes and the Reference Scale. Our investigation involves 4 fundamental Modal Families: *Ionian*, *IpoIonian*, *Harmonic Major* and *Minor* (28 Scales altogether).

The Net Balance between Sharpened and Flattened Degrees (with respect to the Reference Scale) is taken into account: e.g., the *Dorian b2* Mode is obviously darker than the *Dorian*, since the Net Balance between Sharpened/Flattened Degrees is equal to "2 DOWN" for the first Scale, "1 DOWN" for the second (see **Figure** 12).

If the Net Balance happens to be the same, the purest Mode prevails. E.g., *Locrian #6* and *Locrian # 2* (see **Figure** 13) show the same Absolute Brightness (34) and the same Net Balance (3 DOWN): however, the second must be considered as being darker, since it is "Pure" ("Purely Flattened", since there are no Sharpened Degrees with respect to the Reference Scale).

PITCH CLASS SET							SCALE DEGREES UP ↑ and DOWN ↓		SCALE NAME and ABSOLUTE BRIGHTNESS
0	3 ↑	4	6	8 ↑	9 ↑	11 ↑	4 UP (2, 5, 6, 7)		Lydian Augmented #2 (41)
0	2	4	6	8 ↑	9 ↑	11 ↑	3 UP (5, 6, 7)		Lydian Augmented (40)
0	2	4	5 ↓	8 ↑	9 ↑	11 ↑	3 UP (5, 6, 7)	1 DOWN (4)	Ionian Augmented (39)
0	2	4	6	7	8	10	REFERENCE SCALE		Lydian Dominant b6 Mixolydian #4 b6 (37)

Figure 8. Brightness: Modes producing Augmented Seventh Chords.

PITCH CLASS SET							SCALE DEGREES UP ↑ and DOWN ↓		SCALE NAME and ABSOLUTE BRIGHTNESS
0	3 ↑	4	6	7	9 ↑	11 ↑	3 UP (2, 6, 7)		Lydian #2 (40)
0	2	4	6	7	9 ↑	11 ↑	2 UP (6, 7)		Lydian (39)
0	2	4	5 ↓	7	9 ↑	11 ↑	2 UP (6, 7)	1 DOWN (4)	Ionian (38)
0	2	4	5 ↓	7	8	11 ↑	1 UP (7)	1 DOWN (4)	Harmonic Major Ionian b6 (37)
0	2	4	6	7	8	10	REFERENCE SCALE		Lydian Dominant b6 Mixolydian #4 b6 (37)

Figure 9. Brightness: Modes producing Major Seventh Chords.

PITCH CLASS SET							SCALE DEGREES UP ↑ and DOWN ↓		SCALE NAME and ABSOLUTE BRIGHTNESS
0	2	4	6	7	9 ↑	10	1 UP (6)		Lydian Dominant Mixolydian #4 (38)
0	2	4	5 ↓	7	9 ↑	10	1 UP (6)	1 DOWN (4)	Mixolydian (37)
0	2	4	6	7	8	10	REFERENCE SCALE		Lydian Dominant b6 Mixolydian #4 b6 (37)
0	1 ↓	4	5 ↓	7	9 ↑	10	1 UP (6)	2 DOWN (2, 4)	Mixolydian b2 (36)
0	2	4	5 ↓	7	8	10		1 DOWN (4)	Mixolydian b6 (36)
0	1 ↓	4	5 ↓	7	8	10		2 DOWN (2, 4)	Mixolydian b2 b6 (35)

Figure 10. Brightness: Modes producing Dominant Seventh Chords.

PITCH CLASS SET							SCALE DEGREES UP ↑ and DOWN ↓		SCALE NAME and ABSOLUTE BRIGHTNESS
0	2	3 ↓	6	7	9 ↑	11 ↑	2 UP (6, 7)	1 DOWN (3)	Lydian Minor IpoIonian #4 (38)
0	2	4	6	7	8	10	REFERENCE SCALE		Lydian Dominant b6 Mixolydian #4 b6 (37)
0	2	3 ↓	5 ↓	7	9 ↑	11 ↑	2 UP (6, 7)	2 DOWN (3, 4)	IpoIonian (37)
0	2	3 ↓	5 ↓	7	8	11 ↑	2 UP (7)	2 DOWN (3, 4)	Harmonic Minor IpoIonian b6 (36)

Figure 11. Brightness: Modes producing Minor Major Seventh Chords.

PITCH CLASS SET							SCALE DEGREES UP ↑ and DOWN ↓		SCALE NAME and ABSOLUTE BRIGHTNESS
0	2	4	6	7	8	10	REFERENCE SCALE		Lydian Dominant b6 Mixolydian #4 b6 (37)
0	2	3 ↓	6	7	9 ↑	10	1 UP (6)	1 DOWN (3)	Romanian Minor Dorian #4 (37)
0	2	3 ↓	5 ↓	7	9 ↑	10	1 UP (6)	2 DOWN (3, 4)	Dorian (36)
0	1 ↓	3 ↓	5 ↓	7	9 ↑	10	1 UP (6)	3 DOWN (2, 3, 4)	Dorian b2 (35)
0	2	3 ↓	5 ↓	7	8	10		2 DOWN (3, 4)	Aeolian (35)
0	1 ↓	3 ↓	5 ↓	7	8	10		3 DOWN (2, 3, 4)	Phrygian (34)
0	1 ↓	3 ↓	4 ↓↓	7	8	10		3 DOWN (2, 3, 4-)	Super Phrygian (33)

Figure 12. Brightness: Modes producing Minor Seventh Chords.

PITCH CLASS SET							SCALE DEGREES UP ↑ and DOWN ↓		SCALE NAME and ABSOLUTE BRIGHTNESS
0	2	4	6	7	8	10	REFERENCE SCALE		Lydian Dominant b6 Mixolydian #4 b6 (37)
0	2	3 ↓	5 ↓	6 ↓	9 ↑	10	1 UP (6)	3 DOWN (3, 4, 5)	Dorian b5 Locrian #2 #6 (35)
0	1 ↓	3 ↓	5 ↓	6 ↓	9 ↑	10	1 UP (6)	4 DOWN (2, 3, 4, 5)	Locrian #6 (34)
0	2	3 ↓	5 ↓	6 ↓	8	10		3 DOWN (3, 4, 5)	Locrian #2 (34)
0	1 ↓	3 ↓	5 ↓	6 ↓	8	10		4 DOWN (2, 3, 4, 5)	Locrian (33)
0	1 ↓	3 ↓	4 ↓↓	6 ↓	8	10		4 DOWN (2, 3, 4-,5)	Super Locrian (32)

Figure 13. Brightness: Modes producing Half-Diminished Chords.

PITCH CLASS SET							SCALE DEGREES UP ↑ and DOWN ↓		SCALE NAME and ABSOLUTE BRIGHTNESS
0	2	4	6	7	8	10	REFERENCE SCALE		Lydian Dominant b6 Mixolydian #4 b6 (37)
0	1 ↓	3 ↓	5 ↓	6 ↓	8	9 ↓		5 DOWN (2, 3, 4, 5, 7-)	Locrian bb7 (32)
0	1 ↓	3 ↓	4 ↓↓	6 ↓	8	9 ↓		5 DOWN (2, 3, 4-, 5, 7)	Ultra Locrian (31)

Figure 14. Brightness: Modes producing Diminished Seventh Chords.

At this point, the concept of Absolute Brightness can be intuitively extended to Triads. The Absolute Brightness of any Triad is obviously obtained by simply adding up all the Intervals (evaluated with respect to the Root).
We obtain the following hierarchy (the value of Absolute Brightness is written in brackets):

BRIGHTER ↕	Major 3rd (4)	Augmented Triad (12)
		Major Triad (11)
	Minor 3rd (3)	Minor Triad (10)
DARKER		Diminished Triad (9)

Figure 15. Brightness: Triads.

Similarly, we can estimate the Absolute Brightness of Seventh Chords. However, Dominant Seventh and Minor Major Seventh Chords are characterized by the same value of Absolute Brightness (21). In order to deal with this ambiguity, we have to merely notice that the first type of Seventh Chord arises from a Major Triad and the second from a Minor Triad: consequently, we can state that Dominant Seventh Chords are brighter than Minor Major Seventh Chords.

BRIGHTER ↕	Major 3rd (4)	Augmented Triad (12)	Aug, maj7 (23)
		Major Triad (11)	Maj7 (22)
			7 (21)
	Minor 3rd (3)	Minor Triad (10)	Min, maj7 (21)
			Min7 (20)
		Diminished Triad (9)	Min7b5 (19)
DARKER			Dim7 (18)

Figure 16. Brightness: Seventh Chords.

Finally, taking advantage of the hierarchy above (see **Figure 16**), we obtain the underlying Classification (28 Modes involved):

BRIGHTER ↕	Major 3rd (4)	Augmented Triad (12)	Aug, maj7 (23)	Lydian Augmented #2 (41)
				Lydian Augmented – Ionian #4 #5 (40)
				Ionian Augmented – Ionian #5 (39)
		Major Triad (11)	Maj7 (22)	Lydian #2 – Ionian #2 #4 (40)
				Lydian – Ionian #4 (39)
				Ionian (38)
				Harmonic Major – Ionian b6 (37)
			7 (21)	Lydian Dominant – Mixolydian #4 (38)
				Mixolydian (37)
				Mixolydian b2 (36)
				Mixolydian b6 (36)
				Mixolydian b2 b6 (35)
	Minor 3rd (3)	Minor Triad (10)	Min, maj7 (21)	Lydian Minor – IpoIonian #4 (38)
				IpoIonian (37)
				Harmonic Minor – IpoIonian b6 (36)
			Min7 (20)	Romanian Minor – Dorian #4 (37)
				Dorian (36)
				Dorian b2 (35)
				Aeolian – Dorian b6 (35)
				Phrygian – Dorian b2 b6 (34)
				Super Phrygian (33)
		Diminished Triad (9)	Min7b5 (19)	Dorian b5 – Locrian #2 #6 (35)
				Locrian #6 (34)
				Locrian #2 (34)
				Locrian (33)
				Super Locrian – Locrian b4 (32)
			Dim7 (18)	Locrian bb7 (32)
DARKER				Ultra Locrian (31)

Figure 17. Brightness Hierarchy (28 Modes).

4 Final Remarks and Conclusions

In examining the hierarchy in the previous figure, which arises from the analysis herein proposed, several ambiguities and inconsistencies can be easily detected, e.g. the *Mixolydian b2* Scale is brighter than the *Mixolydian b6*, although both the Modes are characterized by an Absolute Brightness (written in brackets) which is equal to 36, and the *Mixolydian b2 b6* is brighter than the *IpoIonian*, although the latter entails a greater Absolute Brightness. The reason is clear: each Degree of a Scale influences the value of Brightness differently. Moreover, the influence of any Degree is not constant, as it depends on the Intervallic Content of the particular Scale. As a consequence, the Brightness of a Mode cannot be estimated by simply adding up all the Components/Elements of the Pitch Class Sect. In the light of what has just been highlighted, in order to effectively remove ambiguities and inconsistencies, we need to resort to a "corrected" relation.
If we denote with B_C the "Corrected" Brightness, we can write:

$$B_C = \sum_{n=2}^{7} k_n s_n$$

As for the coefficients, denoted by k_n, our proposal is as follows:

$$k_2 = 1;\ k_3 = 19;\ k_4 = 2;\ k_5 = 7;\ k_6 = 3;\ k_7 = 8$$

This way, a hierarchy devoid of ambiguities and inconsistencies is finally obtained.

BRIGHTER ↑	Major 3rd (4)	Augmented Triad (12)	Aug, maj7 (23)	Lydian Augmented #2 (262)
				Lydian Augmented – Ionian #4 #5 (261)
				Ionian Augmented – Ionian #5 (259)
		Major Triad (11)	Maj7 (22)	Lydian #2 – Ionian #2 #4 (255)
				Lydian – Ionian #4 (254)
				Ionian (252)
				Harmonic Major – Ionian b6 (249)
			7 (21)	Lydian Dominant – Mixolydian #4 (246)
				Mixolydian (244)
				Mixolydian b2 (243)
				Mixolydian b6 (241)
				Mixolydian b2 b6 (240)
	Minor 3rd (3)	Minor Triad (10)	Min, maj7 (21)	Lydian Minor – IpoIonian #4 (235)
				IpoIonian (233)
				Harmonic Minor – IpoIonian b6 (230)
			Min7 (20)	Romanian Minor – Dorian #4 (227)
				Dorian (225)
				Dorian b2 (224)
				Aeolian – Dorian b6 (222)
				Phrygian – Dorian b2 b6 (221)
				Super Phrygian (219)
		Diminished Triad (9)	Min7b5 (19)	Dorian b5 – Locrian #2 #6 (218)
				Locrian #6 (217)
				Locrian #2 (215)
				Locrian (214)
				Super Locrian – Locrian b4 (212)
↓ DARKER			Dim7 (18)	Locrian bb7 (206)
				Ultra Locrian (204)

Figure 18. Corrected Brightness Hierarchy (28 Modes).

A more general expression may acquire the underlying form:

$$B_C^* = k_0 + \sum_{n=2}^{7} k_n s_n$$

The coefficient k_0 is set, in this case, so as to obtain for the *Ultra Locrian* Mode $B_C^* = B$.

$$k_0 = -173$$

Finally, we obtain the underlying scenario:

BRIGHTER ↑	Major 3rd (4)	Augmented Triad (12)	Aug, maj7 (23)	Lydian Augmented #2 (89)
				Lydian Augmented – Ionian #4 #5 (88)
				Ionian Augmented – Ionian #5 (86)
		Major Triad (11)	Maj7 (22)	Lydian #2 – Ionian #2 #4 (82)
				Lydian – Ionian #4 (81)
				Ionian (79)
				Harmonic Major – Ionian b6 (76)
			7 (21)	Lydian Dominant – Mixolydian #4 (73)
				Mixolydian (71)
				Mixolydian b2 (70)
				Mixolydian b6 (68)
				Mixolydian b2 b6 (67)
	Minor 3rd (3)	Minor Triad (10)	Min, maj7 (21)	Lydian Minor – IpoIonian #4 (62)
				IpoIonian (60)
				Harmonic Minor – IpoIonian b6 (57)
			Min7 (20)	Romanian Minor – Dorian #4 (54)
				Dorian (52)
				Dorian b2 (51)
				Aeolian – Dorian b6 (49)
				Phrygian – Dorian b2 b6 (48)
				Super Phrygian (46)
		Diminished Triad (9)	Min7b5 (19)	Dorian b5 – Locrian #2 #6 (45)
				Locrian #6 (44)
				Locrian #2 (42)
				Locrian (41)
				Super Locrian – Locrian b4 (39)
DARKER ↓			Dim7 (18)	Locrian bb7 (33)
				Ultra Locrian (31)

Figure 19. Alternative Corrected Brightness Hierarchy (28 Modes).

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